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No. 688

SPEED AND PRESSURE RECORDING IN THREE-DIMENSIONAL FLOW

By Dr. F. Krisam

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SPEED AND PRESSURE RECORDING IN THREE-DIMENSIONAL FLOW*

By Dr. F. Krisam

Van der Hegge Zijnen's spherical Pitot tube with its 5 test holes insures a simultaneous record of static pressure and magnitude and direction of velocity in three-dimensional flow. The report treats the method as well as the range of application of this Pitot in the light of modern knowledge on flow around spheres.

1. GENERALITIES

None of the testing devices used heretofore were simple and at the same time accurate enough for precise determination of three-dimensional flow. The standard Pitot tubes of Prandtl, Brabée and others are unsatisfactory for measuring the flow in bends, or back of turbine runners. To be sure, there has been no lack of attempts to obviate this defect by new designs.

As to the history of this spherical Pitot tube, van der Hegge Zijnen himself has written a very detailed report on it.** (Reference 1.) According to this report it is an improvement of Borren's three-hole Pitot. (Reference 2.) Borren in turn had borrowed the idea from Cordier (reference 3) and Ellison (reference 4).

* Über die Messung von Geschwindigkeit und Druck in einer dreidimensionalen Strömung. From Z.F.M., Vol. 23, No. 13, July 14, 1932. pp. 369-373.

** Verbetered instrument ter bepaling van den statischen druk, de grootte en richting der stroomsnelheid van vloeistoffen. De Ingenieur, Vol. 44, No. 38, 1929.

2. DESCRIPTION OF PITOT AND TEST METHODS

The instrument consists of the Pitot sphere a itself, a shaft b and graduated disk c. (Fig. 1.) The 5 pressure holes are arranged on two mutually perpendicular meridians. Scale c is so fastened that its zero point is in the same plane as holes 1, 2, and 3; holes 4, and 5 are spaced exactly alike from hole 2.

In a test the instrument is continuously rotated about shaft b until the manometers hooked up to holes 4 and 5 have exactly the same deflection. Then meridian plane 1, 2, and 3 is in the direction of the flow and angle α between meridian plane 1, 2, 3 and an arbitrarily assumed fixed reference plane can be read on scale 3.

If h_1 , h_2 etc., are the deflections of the manometers connected to the individual pressure holes, then

$$h_1 = p/\gamma + k_1 c^2/2g,$$

$$h_2 = p/\gamma + k_2 c^2/2g \text{ etc.}$$

where p/γ is the pressure within the flow, c the magnitude of the velocity and k_1 , k_2 etc., the coefficients of the individual holes. As to these coefficients, it is assumed for the present that they are independent of the magnitude of the velocity and dependent solely on angle δ , i.e., the angle between flow direction and axis of hole 2. On the basis of this relationship it must be possible to determine angle δ in some way from the different magnitude of the individual test pressures.

Thus

$$h_1 - h_2 = c^2/2g (k_1 - k_2),$$

$$c^2/2g = \frac{h_1 - h_2}{k_1 - k_2} = \frac{h_2 - h_3}{k_2 - k_3} = \frac{h_3 - h_4}{k_3 - k_4} \text{ etc.}$$

and

$$p/\gamma = h_1 - k_1 c^2/2g = h_2 - k_2 c^2/2g \text{ etc.}$$

3. COEFFICIENTS

It follows from the foregoing that the ambit of practical use of the Pitot as regards the coefficients hinges on the following:

- a) that the coefficients obtained are reliable enough;
- b) that the interdependence of the different coefficients on δ lends itself to unequivocal interpretation of δ ;
- c) that the coefficients are unaffected by the magnitude of the velocity within a wide enough margin;
- d) that the surface pressures of the sphere are symmetrically divided with respect to meridian, 1, 2, 3, so that angle α can actually be determined from the equality of the test pressures "5" and "4".

Referring to a, there are two ways of defining the coefficients: test (calibration) and calculus. After all, calibration alone affords reliable figures, because the mathematico-theoretical determination of these coefficients must be carried out under simplifying hypotheses (idealized flow). The potential theory of the frictionless fluid permits the determination of the pressure distribution along the surface of a solid in a flow. For the special case of the sphere (omitting the shaft equation) the calculation becomes especially simple. The superposition of a parallel flow of velocity c_∞ and the flow of a spatial double source yields the velocity potential of the flow round a sphere (reference 5) at:

$$\Phi = c_\infty \times \left(1 + \frac{R^3}{2 r^3}\right)$$

Appropriate differentiation then affords the individual velocity components, and $c^2 = c_x^2 + c_y^2 + c_z^2$ yields the magnitude of the velocity at any point. Our interest, in this case, is confined to the conditions on the surface of the sphere ($r = R$). For it the velocity becomes

$$c_o^2 = c_\infty^2 \left(1 - \frac{x^2}{r^2} \right)$$

and the pressure, according to the Bernoulli equation,

$$p/\gamma_o = p/\gamma_\infty + \frac{c_\infty^2}{2g} \left(\frac{9}{4} \frac{x^2}{r^2} - \frac{5}{4} \right)$$

Herein the term in the brackets is identical with our coefficient k . Putting $\cos \delta' = \frac{x}{r}$, (see fig. 2), yields

$$k = \frac{1}{4} (9 \cos^2 \delta' - 5)$$

For $\delta' = 0$ (forward stagnation point), $k = 1$. The pressure reversal point ($k = 0$) lies at

$$\cos \delta' = \sqrt{\frac{5}{9}}, \text{ i. e. } \delta' \approx 42^\circ$$

This theoretical pressure distribution sets up no sphere resistance in the flow direction, because the pressure distribution is perfectly symmetrical across the front and rear side. In the real fluid the pressure rise on the rear side of the sphere comes only partly into being because of the separation of the boundary layer, and the result is a certain "form resistance" in the direction of flow. On the front side of the sphere, where the influence of the friction is confined to a very thin layer, the actual and the theoretical pressure distribution are amply in accord. Figure 3 shows the theoretical k curve as well as some arrived at by experiment. For comparison the calibration curve of pressure hole 1 was chosen, because on it the disturbing influence of the shaft is least noticeable. In spite of the satisfactory agreement, the calibration of the Pitot can not be foregone for practical use, even if only on account of the mathematically hard to define disturbance of the pressure distribution in the vicinity of the shaft. Figure 4 is a set-up for calibrating the Pitot in a long, straight pipe, as used by the Karlsruhe Institute for flow research. The test station lies in the center of a straight pipe of about 12,000 mm length and 190 mm diameter. For comparison a standard Prandtl Pitot was mounted at the same

station. Both are 35 mm away from the axis; the symmetry of the velocity profile on the test station was checked by repeated measurements. The calibrations were always effected with water. The installation method permitted the adjustment of different angles of flow δ by swinging the whole instrument, and the setting of the manometer deflection $(4 - 5) = 0$ by rotation about the shaft. One of the calibration curves obtained in this manner is reproduced in Figure 5. The shape of curve k_4 follows from the position of hole 4. Holes 4 and 5 each are at about 51° away from hole 2. If $\delta = 0$ at hole 2, it is already $= 51^\circ$ at hole 4, and rises as δ increases. Consequently hole 4 lies always in the "negative pressure zone." This curve k_4 can equally be computed theoretically by the cited method, provided that the somewhat changed angles of flow are taken into account. This calculation could be foregone here because no more than a scattered accord with reality could be expected on account of the consistently very high angles of flow. One can distinctly recognize the additional pressure rise due to the shaft at holes 2 and 3 closest to it. The influence of the shaft has somewhat shifted the whole pressure field toward the direction of $+\delta$. In Figure 6 the manometers are rigged up for calibration. The leads can be flushed from both directions. According to the magnitude of the deflection, the manometers are filled with water or acetylene tetra bromide (specific gravity = 2.98). Reading is by special device, that insures very convenient and exact work.* During calibration the range from $\delta = +60^\circ$ to $\delta = -60^\circ$ was repeatedly covered at 5° intervals, so that there is absolutely no occasion for any accidental error in reading.

Referring to b, with the calibration the coefficients of the individual holes are computed by the agency of quantity p/γ and $c^2/2g$ measured by the standard instrument.

It is

$$k_1 = \frac{2g}{c^2} (h_1 - p/\gamma) \text{ and } k_2 = \frac{2g}{c^2} (h_2 - p/\gamma)$$

The course of the individual curves can be followed in Figure 5. Now the manometer readings h_1 , h_2 etc., are to

* See Krisam-Lorenz: Technical Measurements in the Institute for Flow Machines, by the Technical High School, Karlsruhe. Die Messtechnik, 1930.

reveal a criterion that obtains to the unequivocal definition of angle δ . Such a criterion is the relation

$$\frac{k_3 - k_1}{k_2 - k_4}$$

or for short, k_δ .

k_δ is unequivocally and consistently dependent of δ , and in a range of practically $+60^\circ$ to -60° . The next step is to express k_δ by the manometer deflections, because $k_1, k_2 \dots$ values are determined subsequent to the measurement.

$$k_\delta = \frac{k_3 - k_1}{k_2 - k_4} = \frac{h_3 - h_1}{h_2 - h_4}$$

Referring to c, the paramount assumption on whose justification the applicability of the spherical Pitot stands and falls, is the independence of the coefficients from the magnitude of the velocity. On this subject the resistance curve of the sphere* (reference 6) (see fig. 7) can yield some information. The resistance of the sphere depends on the type of pressure distribution. Consequently, it is readily surmised that this distribution remains the same when the resistance figure of the sphere remains unchanged by different flow velocity. The resistance is practically constant within range of the Reynolds Numbers

$$R = \frac{c \cdot D}{v} = 2 \times 10^4 \text{ to } 1.5 \times 10^5$$

according to Figure 7. This corresponds to an air speed of about 26-200 m/s and a water speed of about 2-16 m/s for a sphere of 12 mm diameter. The upper speed limits are seldom reached, but the low limit seems to be a little high. Still, the variation in resistance coefficient below $R = 2 \times 10^4$ to about 2×10^3 is so small, as to bring the low limit for the constancy of pressure distribution and consequently of the coefficients probably below 1.0 m/s for water and 13 m/s for air with this 12 mm sphere. These conjectures are confirmed by a number of equations and pressure measurements on spheres at different Reynolds Numbers. (See table.)

* Hütte, Vol. I, 25th edition, p. 374.

Experiment by	Medium	Sphere Φ mm	m/s Speed		Reynolds Number	
			min	max	R_{min}	R_{max}
Meyer	air	10	3	30	3.8×10^3	1.9×10^4
v. d. H. Zijnen	"	12	-	20	-	1.5×10^4
Ermisch*	water	20	0.05	-	7.47×10^2	-
		38.2	0.477	0.93	1.75×10^4	2.61×10^4
Karlsruher Institut für Stro- mungsma- schinen	"	12	3.0	4.5	2.9×10^4	4.4×10^4
	"	12	1.0	6.5	9.6×10^3	6.3×10^4
	"	5	1.0	5.5	4×10^3	2.2×10^4

Neither Meyer's nor the Karlsruhe experiments reveal a measurable change in coefficient in spite of the markedly different Reynolds Number. Ermisch, who possibly reached the lowest limit with the 20 mm sphere, studied the two spheres in the same tunnel of only 150 mm height, so that in his experiments the effect of the walls naturally had to be different. But at that, his pressure distribution curves manifest a surprising accord up to about 70° angle of flow, and no deviations occur at angles below that. Besides, it should be remembered that at more than 70° , the pressure hole lies in the eddy zone of the sphere, where an exact measurement is of itself already difficult. Thus Meyer's tests at 70° , also revealed somewhat wider discrepancies from one another at different speeds. The Karlsruhe tests likewise encountered these obstacles at large angles. But there was absolutely no sign of dependence on the speed. Repeated tests at constant speed evinced the same percentage discrepancies of the individual measurements as in a series of tests with different speed. In measurements within this range the difficulties therefore seem to lie in the principle. However, the discrepancies in the individual measurements were not so great as to vitiate a good and accordant mean value by different observers and test programs.

* Reports from the Aerodynamic Institute of the Technical High School, Aachen, No. 6.

So on the basis of these experiments it may be averred that the coefficients of the spherical Pitot are constant for Reynolds Numbers from 3.5×10^3 to beyond 1×10^5 .

Referring to d, pressure holes 4 and 5 are equidistant from hole 2. Angle α is defined through the equality of their pressure records. (Fig. 1.) It therefore is to the interest of a maximum sensitivity of adjustment when holes 4 and 5 lie on a point with maximum pressure gradient. Van d. Hegge Zijnen found in his experiments that the pressure (coefficient) curve had its maximum slope at about 50° , and so arranged holes 4 and 5 accordingly. It is clear that only a symmetrical pressure distribution on the sphere permits its use as measuring instrument in the above sense.

In his pressure measurements on a sphere at different speeds (reference 7) Krell encountered marked dissymmetries. But it seems as if Krell's measured and observed peculiarities (unstable pressure distribution) were primarily due to experimental technique, to which no ordinarily valid character should be ascribed. Within the critical Reynolds Number (see fig. 7) the pressure distribution is unsymmetrical and unstable, and the resistance coefficient drops very abruptly. That the distribution is symmetrical, at least in the subcritical range, can be legitimately assumed according to the reports of various other observers. For instance, there are Ermisch's experiments with very symmetrical pressure distributions in the subcritical range for spheres as well as circular cylinders. This observation on circular cylinders is noteworthy, because here an unsymmetrical distribution is rather expected because of the formation of the Karman vortex street. Eisner's* experiments on circular cylinders are likewise interesting in this respect. He found the distribution symmetrical in the subcritical range, and markedly asymmetrical in the critical range (sudden drop in coefficient of resistance), accompanied by pronounced lateral motions (analogous to Krell's experiments at $c = 50$ m/s). But in the supercritical range the distribution is in principle other than in the subcritical zone, although it is essentially symmetrical again. Eisner measured the pressures on both sides simultaneously.

* Eisner: Resistance Measurements on Cylinders. Mitt. Preuss. Versuchsanstalt für Wasserbau und Schiffbau, Berlin, No. 4, 1929.

Lastly, even the numerous experiments in the Karlsruhe Institute for flow research in water as well as air, failed to show any signs indicative of asymmetrical and unstable pressure conditions on the sphere.

4. ERRORS IN SPHERICAL PITOT MEASUREMENT

In the foregoing it was attempted to prove the particular fitness of this type of Pitot for three-dimensional flow research. The handling is comparatively easy, the range of application ample for practical purposes.

The main difficulties consist in the fact that the measurement is made on a circular surface of a certain size rather than at a point. This fact is, above all, manifest when the flow space to be examined is relatively small. For manufacturing reasons it is hardly possible to make spheres of less than 12 mm. (The manufacturer, the R. Fuess Company, Berlin, tells me of being able to supply spheres of 8 mm diameter.) This size is satisfactory for wind-tunnel study, but not for experiments with water, where the cross sections available are mostly much smaller. The spherical Pitot shares this defect, more or less, with all the other conventional Pitots. On the other hand, the difference between the orderly parallel flow with uniform velocity distribution of definite degree of turbulence in the calibration and the altogether arbitrary character of the test flow is more pronounced in the spherical Pitot than in the other types. The degree of turbulence in the calibration other than in the experiment is an inevitable source of error in all dynamic pressure apparatus. Besides, the spherical Pitot can only be calibrated in a uniform parallel flow, consequently, its use in a flow with marked velocity or pressure gradient

$$\frac{\partial c_x}{\partial y} \text{ or } \frac{\partial p}{\partial y}$$

is an added source of error, which defies evaluation except in especially isolated cases. On top of this is the source of error involved by the wall effect on the coefficients of the sphere in measurements in wall vicinity. Ordinarily, the wall proximity lowers the coefficients of the holes on the half, contiguous to the wall. For the situation in Figure 8, for instance, the greatest effect is at K_3 .

less at K_2 and K_4 . The change in these two is probably identical, so that difference $K_2 - K_4$ is changed little. But $c^2/2g$ is computed from $\frac{h_2 - h_4}{K_2 - K_4}$, so the error induced by wall vicinity is, as a rule, not very great. To compute the pressure in the case of Figure 8, tap 1 is expedient:

$$p/\gamma = h_1 - K_1 c^2/2g$$

The conditions are altogether similar when the other half of the sphere is in wall proximity. Summed up the principal errors involved are:

- 1) Measurement on a relatively large circular surface, not in a point.
- 2) Turbulence different in calibration than in experiment.
- 3) Calibration in steady parallel flow, application in flows with uneven velocity distribution.
- 4) Changed coefficients of measurements in proximity of wall.

However, the greater number of these sources of errors applies equally to any other type of dynamic pressure recording apparatus.

5. MEASUREMENTS WITH SPHERICAL PITOT

In spite of these shortcomings, very satisfactory results can be achieved with this type of Pitot, as seen from the experiments made in the Karlsruhe Institute for flow research. To illustrate: Figure 9 shows the test data in the suction pipe throat of a Kaplan turbine (a high-specific-speed propeller type of turbine with externally adjustable propeller blades). The runner was removed and a number of radial guide vanes took the place of the distributor, thus forming a case of pure meridional flow. There are only two avenues of checking the accuracy of the measurement: firstly, the volume of water computed from the velocity profile must correspond to that measured on the overfall weir; secondly, the total energy at the test station must

approximately equal the total gradient, because the frictional losses are inferior as far as the test station. The radial setting of the Pitot yields the velocity components with the angles of Figure 1.

$$\text{Axial component} \quad c_a = c \cos \delta \sin \alpha$$

$$\text{Peripheral component} \quad c_u = c \cos \delta \cos \alpha$$

$$\text{Radial component} \quad c_r = c \sin \delta$$

The volume of water passing through is

$$Q = 2\pi \int r c_a dr \text{ at } 233.5 \text{ liters/second}$$

whereas the test weir showed 228 liters/second. The discrepancy of about + 2 per cent is slight relative to the size of the Pitot (12 mm by 87 mm test space). The test gradient was 1 m, thus making the total energy = $Q H \gamma = 228 \text{ mkg/s}$. The total energy according to the measurements is:

$$E = 2\pi \gamma \int (p/\gamma + h + c^2/2g) r c_a dr \text{ at } 233.5 \text{ mkg/s}$$

in contrast to an anticipated $E = 228 (1.02)^3 = 241.8 \text{ mkg/s}$ because c_a is already afflicted with 2 per cent error, and this error occurs in the formula in the third power. The difference of about 3.5 per cent between the measured and the theoretical amount of the energy now reverts in part to the friction, and in part to the measuring error when defining the pressure. The cross sections themselves show the existence of a minor c_u component despite the guide vanes. Conformably to the slope of the bounding walls there is a positive c_r component on the outside, and a negative one at the hub. The lag at the hub of the runner, resulting from the increasing cross section becomes manifest in the pressure rise (p/γ) as well as in the flattened velocity profile.

The flow in Figure 9 was still quite orderly, and could have been obtained equally well with standard Pitots. But it is different in Figure 10* which shows the outflow from a Kaplan turbine runner under partial head, measured

* The curves of Figures 9 and 10 are extrapolated for 1 m gradient. The actual test gradient was 4.8 m.

on the same place as the diagrams according to Figure 9. Here the control of the amount of water yields an error of + 0.8 per cent only; quite satisfactory results in spite of the markedly changing velocity across the measuring space. These two examples were taken at random from a great number of experiments, and are no accidental results. (Reference 8.) Such relatively great measuring accuracy is indicative of the possibility of practical elimination of the different errors.

As concerns the practical use of this spherical Pitot we point to manifold control possibility which consists in being able to compute the velocity (and the pressure also) in a different manner, for example:

$$c^2/2g = \frac{h_1 - h_2}{k_1 - k_2} = \frac{h_2 - h_3}{k_2 - k_3} = \frac{h_3 - h_4}{k_3 - k_4} \text{ etc.}$$

In general, the readings on the holes outside of the eddy zone should be used for computing velocity and pressure, because the measurement within the eddy zone is always somewhat less accurate than on the forward half of the sphere.

Translation by J. Vanier,
National Advisory Committee
for Aeronautics

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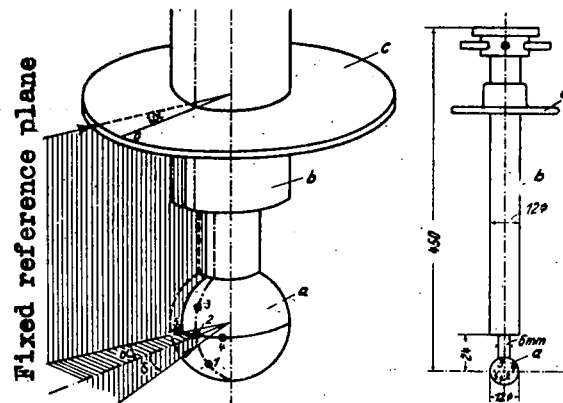


Fig.1
v.d.H.
Zijnen
spherical
pitot.

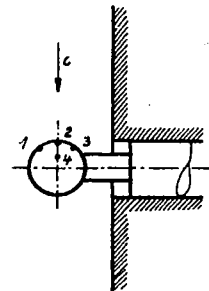
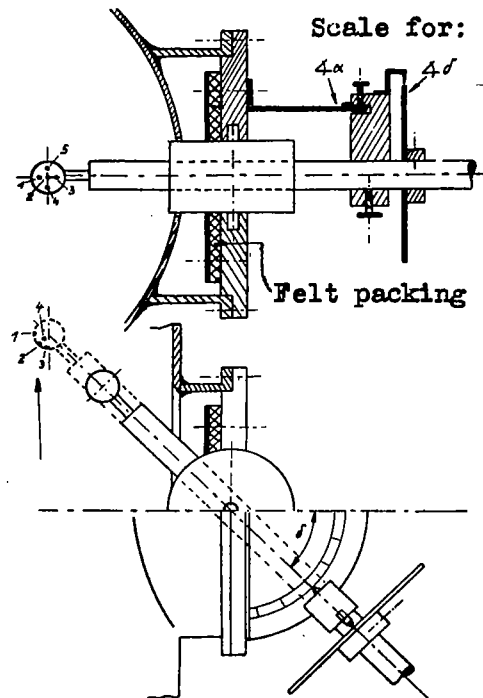


Fig.8

Fig.4
Calibration
set
up.

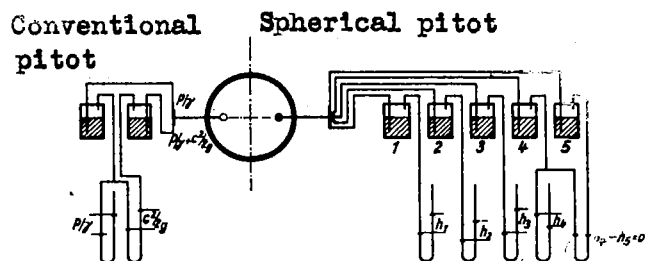


Fig.6 Manometers rigged up for calibration.

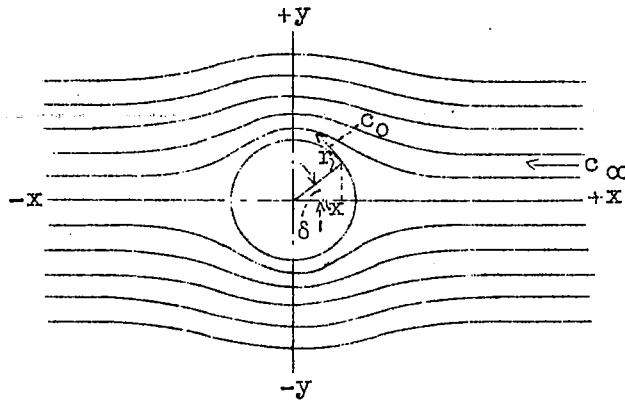
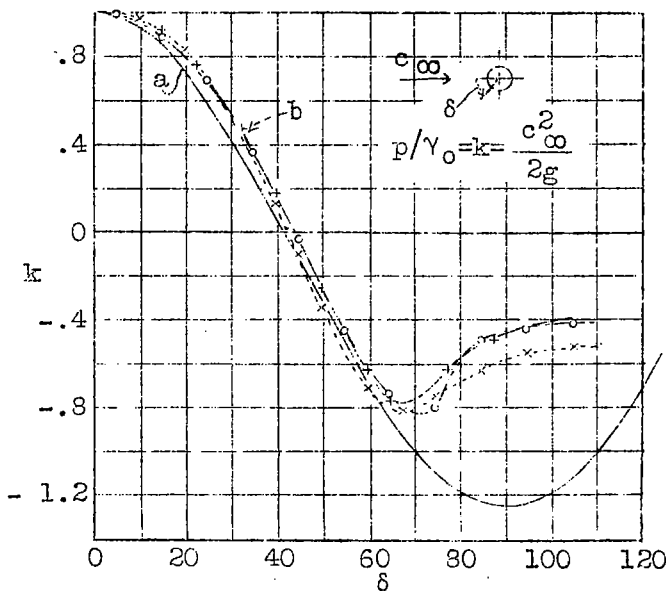


Fig.2 Flow round a sphere.



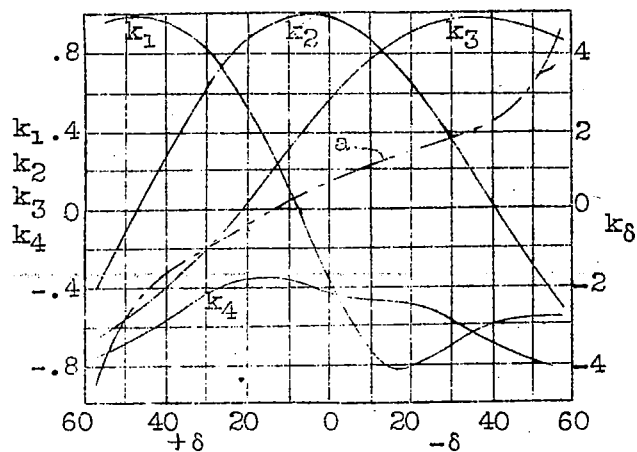
a, Potential pressure distribution.

b, Measured pressure.

Fig.3 Theoretical and actual pressure distribution on the sphere.

$$a, k_\delta = \frac{k_3 - k_1}{k_2 - k_4}$$

Fig.5 Calibration curves.



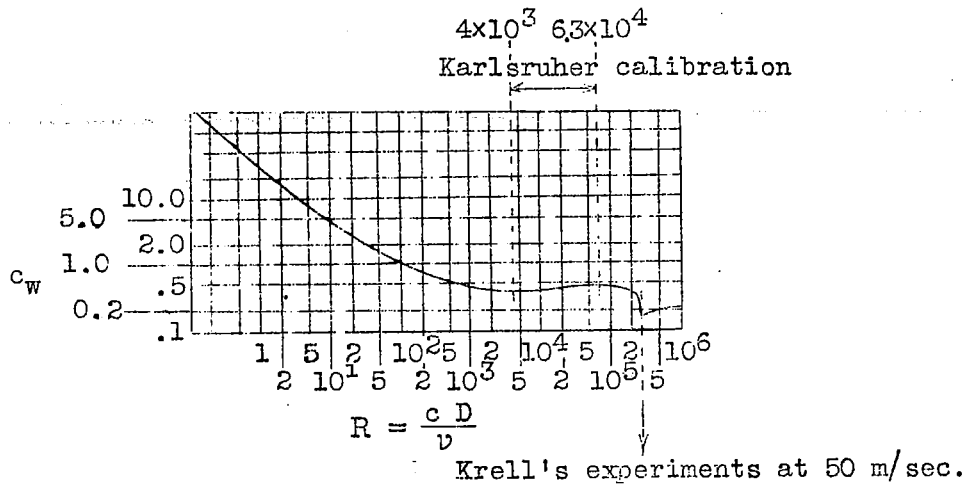


Fig.7 Coefficient of resistance of sphere.

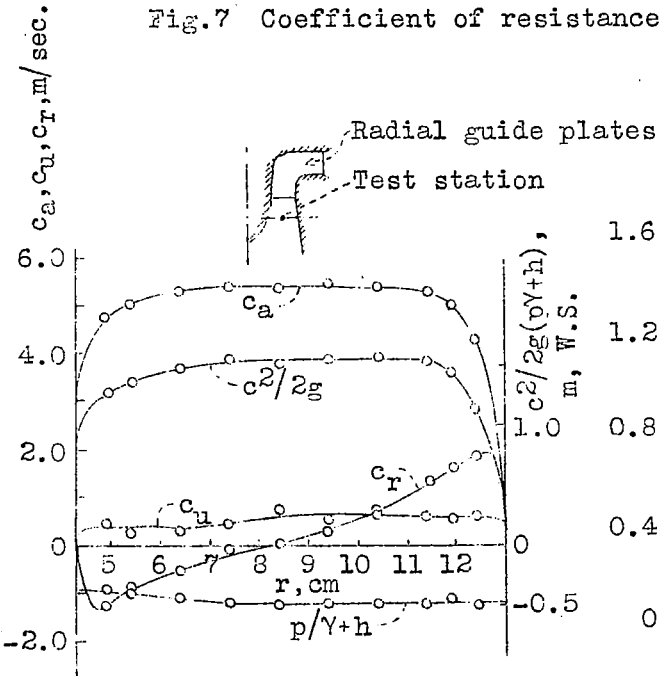


Fig.9 Pressure and velocity distribution of a meridional flow.

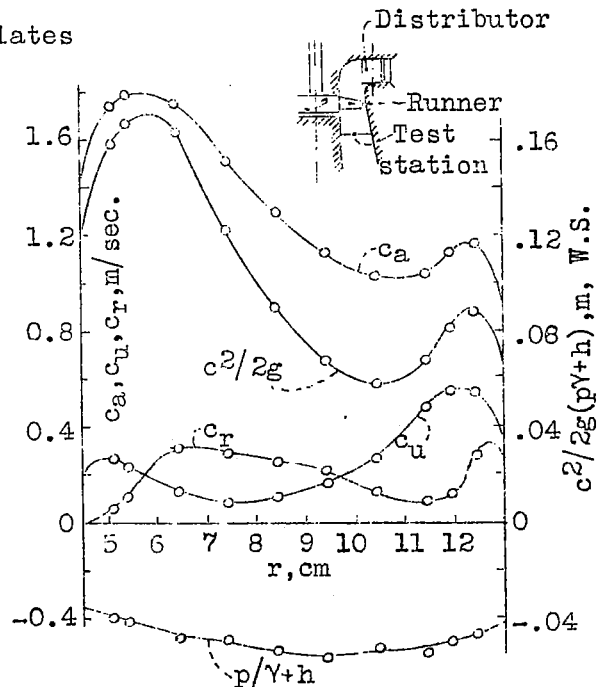


Fig.10 Pressure and velocity distribution behind a turbine runner.